6.7 Applications of Polynomial Equations

Objectives

1) Solve word problems using direct translation
2) Solve word problems using geometry formulas
3) Solve word problems with given functions
4) Solve word problems using the Pythagorean Theorem.

* There are several errors on the handout *

Exam #2 date + review + topics

Selected Review Problems

Math 70: 6.7 Application Problems with Factoring Process

Set up and solve.

1) The product of two consecutive integers is 156. Find the integers.

errors (1) (2)

2) The area of a triangle is 320 square inches. The height is 12 inches longer than it is wide. What are the dimensions of the triangle?

(3)

3) A valuable sports card is 4 cm wide and 5 cm long. The card is to be sandwiched by two pieces of Lucite, each of which is 5.5 times the area of the card. Determine the dimensions of the Lucite that will ensure a uniform border around the card.

4) The diagonal of a rectangle is 50 feet. The length is 10 feet longer than the width. What are the dimensions of the rectangle?

(4)

5) The force in newtons needed to stretch a certain spring x centimeters from its resting position is given by the polynomial function $f(x) = 3x^2$ where 3 is the spring constant. If a force of 48 newtons is applied, how far will the spring be stretched?

6) A certain rectangle's length is 5 feet longer than its width. If the area of the rectangle is 66 square feet, find its dimensions.

7) Find the length of the shorter leg of a right triangle if the longer leg is 24 meters and the hypotenuse is 6 more than twice the shorter leg.

(5)

8) A window washer accidentally drops a bucket from the top of a 64-foot building. The height h of the bucket after t seconds is given by $h(t) = -16t^2 + 64$. When will the bucket hit the ground?

Math 70

① The product of two consecutive integers is 156.

product → multiply; two → two values/expressions; consecutive integers → numbers w/o decimal part in a row → x and $(x+1)$; is → equal

$x \cdot (x+1) = 156$

$x^2 + x = 156$ distribute

$x^2 + x - 156 = 0$ set = 0

$(x+13)(x-12) = 0$ factor

-156: $+13 \times -12$, 1

$x = -13 \quad x = 12$

One pair of consecutive numbers $x = -13$

$x+1 = -13+1 = -12$

-13 and -12

Another pair of consecutive numbers $x = 12$

$x+1 = 12+1 = 13$

12 and 13

② The area of a triangle is $320\,in^2$. The height is 12" longer than the base.

area of a triangle → $A = \frac{1}{2}B \cdot H$; is → equals; height → H; is → $=$; 12" longer than the base → $12 + B$

$A = \frac{1}{2}BH = 320$

... wide. What are the dimensions.

wide → B; dimensions → B and H

$\frac{1}{2}B(12+B) = 320$ mult by $\frac{1}{2}$ to clear fractions

$B(12+B) = 640$ dist

$12B + B^2 = 640$ set = 0.

$B^2 + 12B - 640 = 0$ factor

-640: 32×-20, 12

$(B+32)(B-20) = 0$

cont...

Math 70

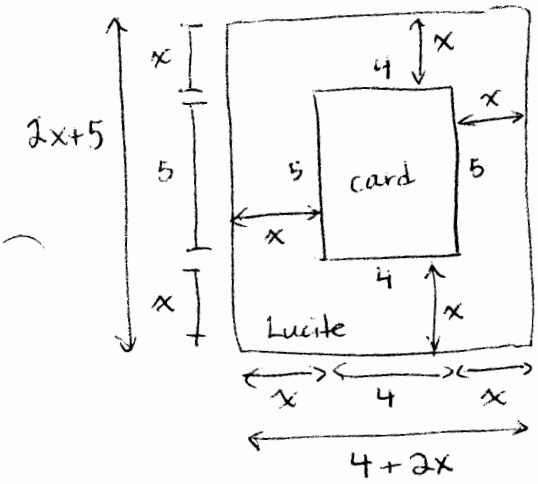
$B = -32$ extraneous since lengths can't be negative

$\boxed{B = 20 \text{ in}}$

$H = B + 12$

$H = 20 + 12 = \boxed{32 \text{ in} = H}$

(3) A sports card is 4cm wide and 5cm long. is to be sandwiched between two pieces of Lucite 5.5 times the area of the card. Find dimensions of Lucite that ensure a uniform border



Area of card $= L \cdot W = 4 \cdot 5 = 20 \text{ cm}^2$

Area of Lucite $= (5.5)(20) = 110 \text{ cm}^2$

uniform border $\Rightarrow$ same width x on all four sides.

Area of Lucite $= L \cdot W = 110$

$(2x+4)(2x+5) = 110$ subst for L & W

$4x^2 + 10x + 8x + 20 = 110$ FOIL

$\frac{4x^2}{2} + \frac{18x}{2} - \frac{90}{2} = \frac{0}{2}$ set = 0

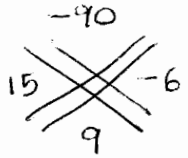
$2x^2 + 9x - 45 = 0$

$2x^2 + 15x - 6x - 45 = 0$

$x(2x+15) - 3(2x+15) = 0$

$(2x+15)(x-3) = 0$

$x = -\frac{15}{2}$ extraneous $x = 3 \Rightarrow$ width of border



1, 90
2, 45
3, 30
5, 18
6, 15

cont...

Math 70

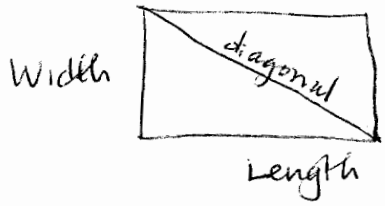
dimensions of Lucite : subst $x=3$

$W = 4+2x \Rightarrow 4+2(3) = 4+6 = 10$

$L = 5+2x \Rightarrow 5+2(3) = 5+6 = 11$

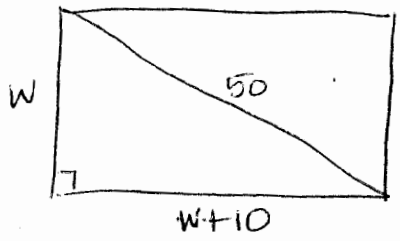
dimensions $\boxed{10\text{ cm} \times 11\text{ cm}}$

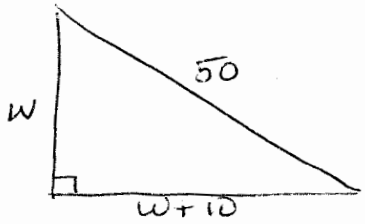
④ The diagonal of a rectangle is (↓ equal) 50 feet.



The length is 10 ft longer than width. Find dimensions.

L equals 10 + W $\Rightarrow$ $L = 10 + W$





use right triangle only

Pythagorean Theorem for sides of a right triangle

$A^2 + B^2 = C^2$

$W^2 + (W+10)^2 = 50^2$ substitute

$W^2 + (W+10)(W+10) = 2500$ FOIL

$W^2 + W^2 + 20W + 100 = 2500$ Set = 0.

$\frac{2W^2}{2} + \frac{20W}{2} - \frac{2400}{2} = \frac{0}{2}$ divide all both sides by 2

$W^2 + 10W - 1200 = 0$

$(W-30)(W+40) = 0$

$W = 30$ $\quad$ $W = -40$ extraneous, lengths can't be negative.

-1200	
-30	40
10	

$-10, 120$

$-20, 60$

$-30, 40$

$\boxed{W = 30 \text{ feet}}$ $\quad$ $L = W + 10 = 30 + 10 = \boxed{40 \text{ feet} = L}$

Math 70

(5) Force needed to stretch x cm is $f(x) = 3x^2$ (force). If a force of 48 Newtons is applied, how far stretched? (subst 48 for f(x))

$48 = 3x^2$

$\frac{3x^2}{3} - \frac{48}{3} = \frac{0}{3}$

$x^2 - 16 = 0$ diff of sq

$(x-4)(x+4) = 0$

$x = 4$ cm or $x = -4$ extraneous

(6) A certain rectangle's length (L) is 5 ft longer than ($5+$) width (W). If the area is 66 ft^2, find dimensions.

$L \cdot W = 66$ area formula

$L = 5 + W$ subst

$(5+W)W = 66$ dist

$5W + W^2 = 66$ set = 0

$W^2 + 5W - 66 = 0$

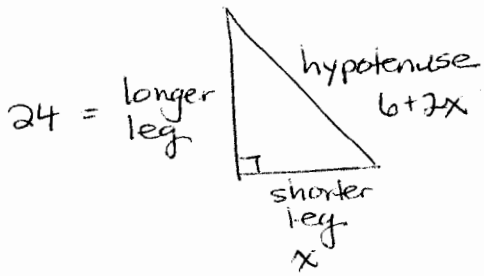
$(W-6)(W+11) = 0$

-66: $-6 \cdot 11$, $-6 + 11 = 5$

$W = 6$ $W = -11$ extraneous

$W = 6$ ft $L = W + 5 = 6 + 5 =$ 11 ft $= L$

(7) Find the length of the shorter leg of a right triangle if the longer leg is 24 meters and the hypotenuse is 6 more than twice the shorter leg.



Pythagorean Theorem

$A^2 + B^2 = C^2$

$x^2 + 24^2 = (6+2x)^2$

$x^2 + 576 = 36 + 24x + 4x^2$

$0 = 3x^2 + 24x - 540$

cont...

Math 70

$$\frac{3x^2}{3} + \frac{24x}{3} - \frac{540}{3} = \frac{0}{3}$$

$$x^2 + 8x - 180 = 0$$

$$(x+18)(x-10) = 0$$

-180: $18 \cdot -10$, $18 + (-10) = 8$

$x = -18$
extraneous

$x = 10$ m
shorter leg

(8) A window washer accidentally drops a bucket from the top of a 64-foot building. The height h of the bucket after t seconds is given by $h(t) = -16t^2 + 64$. When will the bucket hit the ground?

↓
height = 0
Subst 0 for $h(t)$

$$\frac{0}{-16} = \frac{-16t^2}{-16} + \frac{64}{-16}$$

$$0 = t^2 - 4 \qquad \text{diff of sq.}$$

$$(t-2)(t+2)$$

$t = 2$ sec

$t = -2$
extraneous

Math 70 EXAM #2 Topics List

Math 70-33: Exam #2 Wednesday, 14 March, 1:20 PM – 2:25 PM
ath 70-34: Exam #2 Tuesday, 13 March, 1:20 PM – 2:25 PM

Optional Exam 2 Review session / office hours for any Math 70 student in either section:
Friday, March 9, 3:30 PM – 10AM – 12 noon, room 342.

Martin-Gay		Your book: Bittinger
4.1 & 4.4	10	4.1, 4.2, 4.3 & 9.3-1st
4.4 & 4.2	11	9.3-2nd & 9.1
4.3	12	4.4
	13	(no lesson – day of exam 1)
4.3 & 3.1	14	9.2 & 4.5-1st
3.1, 5.4, +	15	4.5-2nd, 5.5, & 5.6*
6.4	16	5.8
9.1, 5.5, 5.6	17	5.9, 6.1 & 6.2
5.6 & 5.7	18	6.3, 6.4 & 6.5
factoring process	19	6.6*
5.8	20	6.7

If you are repeating Math 70 or study with students in other sections of Math 70, it may be helpful to know how the other textbook, Martin-Gay, aligns with your textbook, Bittinger.

Math 70 Exam 2 Topics List (Lesson 20)

chapter 4 Linear Systems in 2 variables and

- 4.1 Solving systems by graphing
  classifying systems
- 4.2 Solving systems by substitution
- 4.3 Solving systems by elimination

chapter 9 Linear Systems in 3 variables

- 9.3 Solving systems using matrices on GC
- 9.1 Solving systems in 3 variables
- 4.4 Solving applications of 2x2 systems
- 9.3 Solving applications of 3x3 systems
- 4.5 Solving equations by graphing
  - intersection method
  - x-intercept method

chapter 5 Polynomials

- 5.5 Multiplying polynomials
  - distribute
  - FOIL
  - polynomial x polynomial
- 5.6 Special products
  - difference of squares
  - perfect square trinomials
  - perfect cube polynomials
  - difference of cubes
  - sum of cubes
- 5.8 Division of polynomials
  - by a monomial
  - by a polynomial
  - long division
  - synthetic division
- 5.9 The Algebra of Functions
  sum, difference, product, quotient

Exam 2 Topics, continued

Chapter 6 Factoring + Solving Equations by Factoring

6.1 Greatest Common Factor
Grouping
Zero-product property

6.2 Trinomials x^2+bx+c

6.3 Trinomials ax^2+bx+c $a \neq 1$, a not a GCF

6.4 Perfect Square Trinomials
Difference of Squares
Sum of Squares

6.5 Sum of Cubes
Difference of Cubes

6.6. A General Strategy for Factoring

6.7 Solving applications

Math 70: Selected Review Questions for Exam #2

Solve the system of equations using matrices. Write the input and output matrices, then write the solution as an ordered pair or ordered triple, the empty set, or set notation showing an infinite number of solutions.

1) $\begin{cases} x+y=3 \\ 2y=10 \\ 3x+2y-3z=1 \end{cases}$

2) $\begin{cases} 2x+y=-2 \\ 2x-z=3 \\ y+z=-5 \end{cases}$

3) $\begin{cases} x+2y-z=5 \\ 6x+y+z=7 \\ 2x+4y-2z=5 \end{cases}$

4) $\begin{cases} 5x+y=-5 \\ 5x-z=5 \\ y+z=-10 \end{cases}$

Solve the equation graphically and round the solution to the nearest hundredth, if necessary. Identify which method you used and the equation(s) you graphed.

5) $1.1x+4.1=0.5x+0.68$

6) $\frac{1}{3}(6x-9)=6\left(\frac{1}{3}x-\frac{1}{2}\right)+6$

7) $0.06(4x-3)=0.24(x+7)-1.86$

8) $3x+\sqrt{11}=6x-\sqrt{2}$

9) $8\pi x+1=2.7(x+\pi)$

Solve the problem.

10) ABC Phone Company charges \$24 per month plus 8 cents per minute of long distance. XYZ Phone Company charges \$18 per month plus 12 cents per minute of long distance. If x represents the number of minutes, then the month cost for service can be modeled by the equations

$$\begin{cases} C_1=24+0.08x \\ C_2=18+0.12x \end{cases}$$

Which phone company's cost is lower if 30 minutes of long distance are used? How many minutes of long distance would make the two companies' costs the same?

11) Megan is considering a job in sales. She is offered a monthly salary of \$1500 plus 4% commission with one company or a 9% commission-only position. How much must she sell per month, in order that the two plans give her the same gross pay?

12) Carole has a 20% alcohol solution and a 80% alcohol solution. Find how many liters of each solution she should mix to make 22 liters of a 35% alcohol solution.

Math 70 Exam 2 Selected Review

① $\begin{cases} x+y=3 \\ 2y=10 \\ 3x+2y-3z=1 \end{cases}$

$\Rightarrow \begin{cases} x+y+0z=3 \\ 0x+2y+0z=10 \\ 3x+2y-3z=1 \end{cases}$

$\Rightarrow \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 2 & 0 & 10 \\ 3 & 2 & -3 & 1 \end{bmatrix}$

$\Rightarrow$ RREF $\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ $\Rightarrow$ $\boxed{(-2, 5, 1)}$

② $\begin{cases} 2x+y=-2 \\ 2x-z=3 \\ y+z=-5 \end{cases}$

$\Rightarrow \begin{cases} 2x+y+0z=-2 \\ 2x+0y-z=3 \\ 0x+y+z=-5 \end{cases}$

$\Rightarrow \begin{bmatrix} 2 & 1 & 0 & -2 \\ 2 & 0 & -1 & 3 \\ 0 & 1 & 1 & -5 \end{bmatrix}$

$\Rightarrow$ RREF $\begin{bmatrix} 1 & 0 & -1/2 & 3/2 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\Rightarrow$ $\boxed{\{(x,y,z):\ x-\frac{1}{2}z=\frac{3}{2} \text{ and } y+z=-5\}}$

(3) $\begin{cases} x + 2y - z = 5 \\ 6x + y + z = 7 \\ 2x + 4y - 2z = 5 \end{cases}$

$\Rightarrow \begin{bmatrix} 1 & 2 & -1 & 5 \\ 6 & 1 & 1 & 7 \\ 2 & 4 & -2 & 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 0 & 3/11 & 0 \\ 0 & 1 & -7/11 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\Rightarrow$ **no solution** or $\boxed{\phi}$

(4) $\begin{cases} 5x + y = -5 \\ 5x - z = 5 \\ y + z = -10 \end{cases}$

$\Rightarrow \begin{cases} 5x + y + 0z = -5 \\ 5x + 0y - z = 5 \\ 0x + y + z = -10 \end{cases}$

$\Rightarrow \begin{bmatrix} 5 & 1 & 0 & -5 \\ 5 & 0 & -1 & 5 \\ 0 & 1 & 1 & -10 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 0 & -1/5 & 1 \\ 0 & 1 & 1 & -10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\Rightarrow$ $\boxed{\{(x,y,z);\ x - \frac{1}{5}z = 1 \text{ and } y + z = -10\}}$

(5) $1.1x + 4.1 = 0.5x + 0.68$

intersection

$y_1 = 1.1x + 4.1$

$y_2 = 0.5x + 0.68$

$(-5.7, -2.17)$

solution $\boxed{x = -5.7}$

⑥ $\frac{1}{3}(6x-9) = 6\left(\frac{1}{3}x - \frac{1}{2}\right) + 6$

Intersection

$y_1 = (6x-9)/3 \rightarrow \frac{6}{3}x$

$y_2 = 6(x/3 - 1/2) + 6 \rightarrow \frac{6}{3}x$

same slopes!

Parallel lines $\Rightarrow$ **no solution** or **$\emptyset$**

⑦ $0.06(4x-3) = 0.24(x+7) - 1.86$

intersection

$y_1 = 0.06(4x-3)$

$y_2 = 0.24(x+7) - 1.86$

same line $\Rightarrow$ infinitely many solutions

all real #s

⑧ $3x + \sqrt{11} = 6x - \sqrt{2}$

intersection

$y_1 = 3x + \sqrt{11}$

$y_2 = 6x - \sqrt{2}$

$x = 1.5769461 \quad y = 8.0474631 \Rightarrow$ **(1.58, 8.05)**

⑨ $8\pi x + 1 = 2.7(x+\pi)$

Intersection

$y_1 = 8\pi x + 1$

$y_2 = 2.7(x+\pi)$

$x = .33354373 \quad y = 9.3828682 \Rightarrow$ **(.33, 4.38)**

(10) $\begin{cases} y = 24 + 0.08x & ABC \\ y = 18 + .12x & XYZ \end{cases}$

30 mins $\Longrightarrow$ ABC $y = 24 + 0.08(30) = \$26.40$

XYC $y = 18 + .12(30) = \$21.60$

XYZ is less expensive for 30 minutes

Solve system — by graphing, matrices, subst, elimination

$\begin{cases} y = 24 + 0.08x \\ y = 18 + .12x \end{cases}$

Substitution:

$18 + .12x = 24 + 0.08x$

$.04x = 6$

$x = 150$ min

(11) 1st company $\begin{cases} y = 1500 + .04x \end{cases}$ where x = amount of sales

2nd company $\begin{cases} y = .09x \end{cases}$

Solve system — by graphing, matrices, subst, elimination

Substitution

$0.09x = 1500 + 0.04x$

$0.05x = 1500$

$x = \$30,000$ in sales

(12) combine 20% and 80% solutions to get 22 L of 35%.

x y

$\begin{cases} x + y = 22 \\ .2x + .8 = .35(22) \end{cases}$

continued.

Solve System by graphing, substitution, elimination, or matrices.

Matrices

$$\begin{bmatrix} 1 & 1 & 22 \\ .2 & .8 & 7.7 \end{bmatrix}$$

RREF $\Rightarrow$

$$\begin{bmatrix} 1 & 0 & 16.5 \\ 0 & 1 & 5.5 \end{bmatrix}$$

$x =$ 16.5 L of 20% solution

$y =$ 5.5 L of 80% solution